

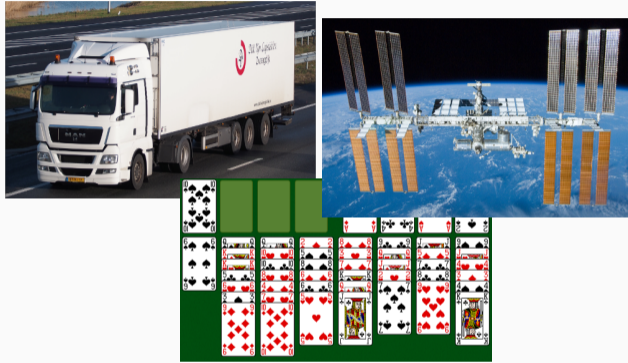
Sensitivity Analysis for Saturated Post-hoc Optimization

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Optimal Classical Planning



- Find cost-optimal action sequence that achieves goal
- Deterministic, fully observable, domain-independent
- A* search with **admissible** heuristic

Abstractions

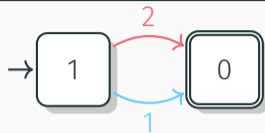
Abstraction

Simplified statespace from abstraction
function $\alpha : S \rightarrow S^\alpha$

Use multiple different abstractions:

$H = \langle h_{\alpha_1}, \dots, h_{\alpha_n} \rangle$ as heuristic

Addition of abstraction heuristics **not**
admissible

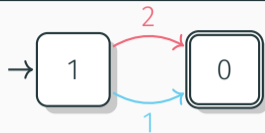


Cost Partitioning

Additive Abstractions

$H = \langle h_1, \dots, h_n \rangle$ is additive if $\sum_i^n cost_i(o) \leq cost(o)$ for all $o \in O$

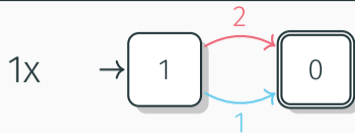
Redistribute costs for abstractions: Cost partitioning



PhO Heuristic

Scale abstractions to obey additivity constraints:

$$\sum_i^n \text{cost}_i(o) \leq \text{cost}(o) \text{ for all } o \in O$$



PhO Heuristic

Scale abstractions to obey additivity constraints:

$$\sum_i^n \text{cost}_i(o) \leq \text{cost}(o) \text{ for all } o \in O$$



PhO Heuristic

H a set of abstraction
heuristics h

$h(s)$ abstract goal distance of
state s in abstraction h

$cost_h(o)$ cost of operator o in
heuristic h

o affects h operator appears on a
state-changing edge in h



PhO LP for state s

$$\begin{aligned} & \text{maximize } \sum_{h \in H} h_i(s) \cdot w_h \text{ s.t.} \\ & \sum_{h \in H: o \text{ affects } h} \text{cost}(o) \cdot w_h \leq \text{cost}(o) \text{ for all } o \in O \\ & w_h \geq 0 \text{ for all } h \in H \end{aligned}$$

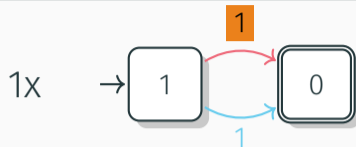
SPhO Heuristic

SPhO Linear Program

minimize $\sum_{o \in O} \text{cost}(o) \cdot Y_o$ s.t.

$\sum_{o \in O} \text{mscf}_h(o) \cdot Y_o \geq h(s)$ for all $h \in H$

$Y_o \geq 0$ for all $o \in O$



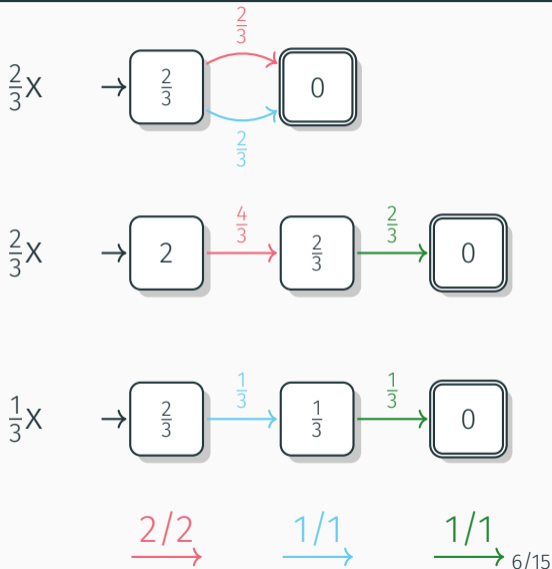
SPhO Heuristic

SPhO Linear Program

$$\text{minimize } \sum_{o \in O} \text{cost}(o) \cdot Y_o \text{ s.t.}$$

$$\sum_{o \in O} \text{mscf}_h(o) \cdot Y_o \geq h(s) \text{ for all } h \in H$$

$$Y_o \geq 0 \text{ for all } o \in O$$



- LP computations expensive
- cost scaling cheap to evaluate

→ reduce LP computations by re-using cost partitionings

h_s^{SPhO} only optimized for state s

$$h_s^{SPhO}(s') \leq h_{s'}^{SPhO}(s')$$

Cover Rule

h_s^{SPhO} covers s' if $h_s^{SPhO}(s') = h_{s'}^{SPhO}(s')$

no approximation

$Sols \leftarrow \emptyset$

function LAZY_SPHO(s)

if COVERS(sol, s) any $sol \in Sols$ **then**

return ADAPT(sol, s)

$\langle sol, h \rangle \leftarrow$ solve SPhO LP for s

$Sols \leftarrow Sols \cup \{sol\}$

return h

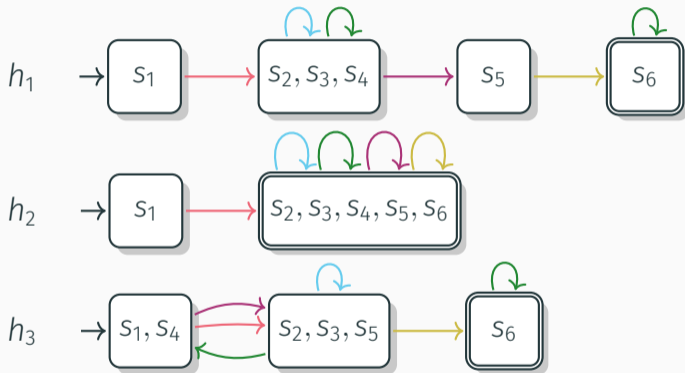
Example

SPhO Constraints:

$$Y_{01} + Y_{04} + Y_{05} \geq h_1(s)$$

$$Y_{01} \geq h_2(s)$$

$$Y_{01} + Y_{04} + Y_{05} \geq h_3(s)$$



Cover Rule: Grouped

$$Y_{01} + Y_{04} + Y_{05} \geq \max(h_1(s), h_3(s))$$

$$Y_{01} \geq h_2(s)$$

	s_1	s_2	s_3	s_4	s_5
$\max(h_1, h_3)$	3	2	2	2	1
h_2	1	0	0	0	0

Cover Rule: Grouped

h_s^{SPHO} covers s' if $\langle H_1(s), \dots, H_m(s) \rangle = \langle H_1(s'), \dots, H_m(s') \rangle$

Sensitivity Analysis

$$\begin{aligned} &\text{minimize } Y_{01} + Y_{03} + Y_{04} + Y_{05} \text{ st} \\ &Y_{01} + Y_{04} + Y_{05} \geq \max(h_1(s), h_3(s)) \\ &Y_{01} \geq h_2(s) \end{aligned}$$

Optimal basis still optimal after changes
→ Sensitivity Analysis

Cover Rule: 100%

	s_1	s_2	s_3	s_4	s_5
$\max(h_1, h_3)$	3	2	2	2	1
h_2	1	0	0	0	0

	SA	
$h(s_1)$	L	U
$\max(h_1, h_3) = 3$	1	∞
$h_1 = 1$	$-\infty$	3

Cover Rule: 100%

h_s^{SPhO} covers s' if the $\sum_i^m \Delta H_j \leq 1$

Generalization of all other rules

Results

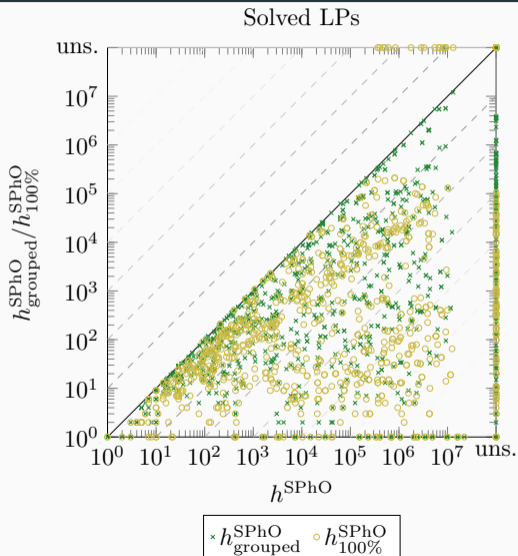
Tasks optimal IPC tasks from
1998-2018

Time 30min

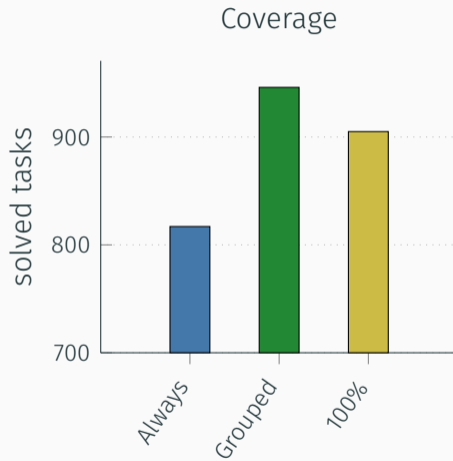
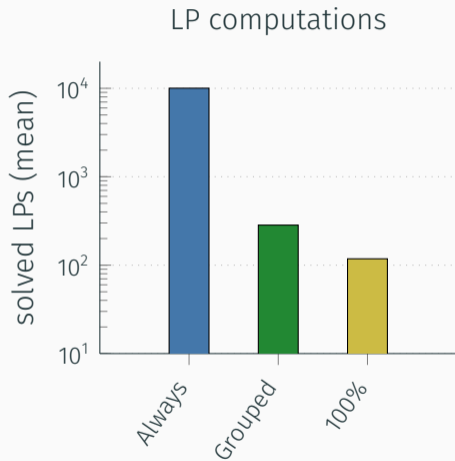
Memory 8G

Abstractions systematic PDBs size 2

LP Solver CPLEX



Results



Grouped: $O(1)$, Percent: $O(n)$

Lazy SPhO:

- Bound storage size for LPs → only consider most recent or most useful

Sensitivity Analysis:

- Apply to other structurally fixed heuristics: Potential Heuristics
- Sensitivity Analysis possible on rows or columns of the constraint matrix

LP solutions give other useful information: Reduced Cost, basic constraints, ...

- Identify useful patterns
- Abstraction refinement