

From Non-Negative to General Operator Cost Partitioning

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January 29, 2015

Introduction

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State space search

- Common approach: A^* with admissible heuristic
- One heuristic often not sufficient
- How to combine heuristics?

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 - Sum? Not admissible
 - Maximum? Does not use all information

Breakthrough: Cost partitioning

- Make arbitrary heuristics additive
- Part of many state-of-the-art heuristics

Operator Cost Partitioning

Main idea

- Create **copies** of the original problem
- **Distribute operator cost function** between copies
- Compute one heuristic per copy
- Sum resulting heuristic values

Operator Cost Partitioning

Operator Cost Partitioning [Katz and Domshlak 2010]

Find cost functions c_1, \dots, c_n with

- Non-negative costs: $c_i \geq 0$
- Costs are distributed: $\sum_i c_i \leq \text{original cost}$

⇒ Admissible estimates using cost function c_i are additive

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Why restrict costs to non-negative values?

General Operator Cost Partitioning

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General Operator Cost Partitioning

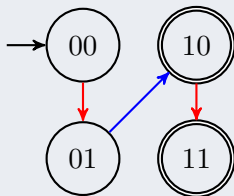
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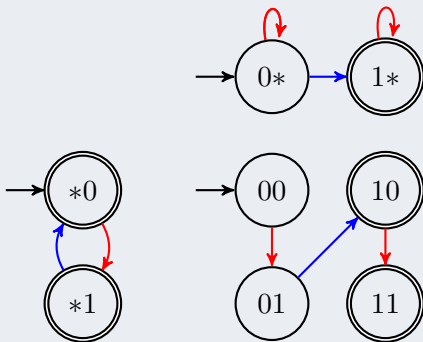
General Cost Partitioning Example

Example



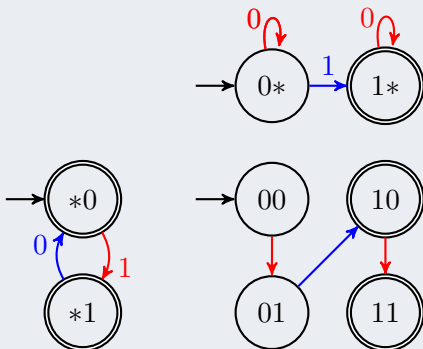
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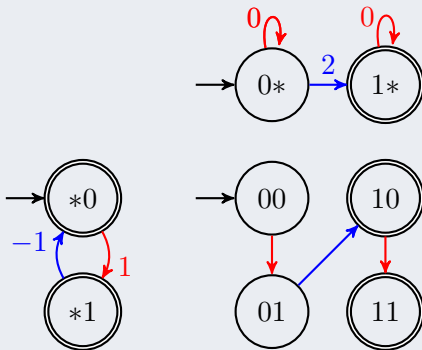
Example



Heuristic value: $0 + 1 = 1$

General Cost Partitioning Example

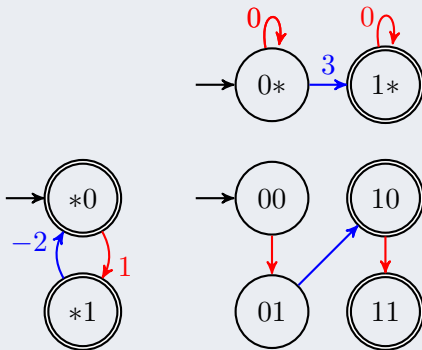
Example



Heuristic value: $0 + 2 = 2$

General Cost Partitioning Example

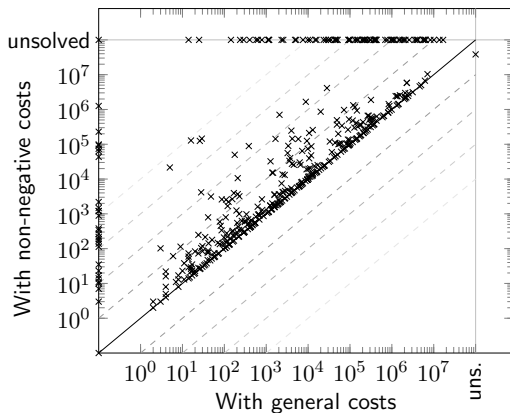
Example



Heuristic value: $-\infty + 3 = -\infty$

Heuristic Quality of General Cost Partitioning

Expansions for optimal cost partitioning of atomic projections



Relation to Other Topics in Heuristic Search Planning

General Operator Cost Partitioning in Relation to ...

- Operator-counting heuristics
- State equation heuristic
- A new approach to heuristic construction (potential heuristics)

1) Operator-Counting Heuristics

Operator-counting heuristics [Pommerening et al. 2014]

- Minimize total plan cost
- Subject to necessary properties of any plan (constraints)

Different sets of constraints define different heuristics

1) Operator-Counting Heuristics: Theoretical Result

Theorem

Combining **operator-counting heuristics** in one LP
is equivalent to
computing their **optimal general cost partitioning**.

2) State Equation Heuristic

Special case: state equation heuristic [van den Briel et al. 2007, Bonet 2013]

- Categorization previously unclear
 - Landmarks?
 - Abstractions?
 - Delete relaxations?
 - Critical paths?

2) State Equation Heuristic

Special case: state equation heuristic [van den Briel et al. 2007, Bonet 2013]

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Theorem

State equation heuristic

=

Optimal general cost partitioning of all atomic projection heuristics

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Theorem

State equation heuristic

=

Optimal general cost partitioning of all atomic projection heuristics

3) Potential Heuristics

Potentials

- Numerical value associated with each fact
- Heuristic value is **sum of potentials** for facts in state



Image credit: David Lapetina

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Linear constraints over potentials

- Express **consistency** and **admissibility**
- **Necessary and sufficient** conditions



Image credit: David Lapetina

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Potentials

- Numerical value associated with each fact
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Linear constraints over potentials

- Express **consistency** and **admissibility**
- **Necessary and sufficient** conditions

Optimization criterion

- Can optimize **any function** over potentials
- Here: maximize heuristic value of a state



Image credit: David Lapetina

3) Potential Heuristics: Theoretical Result

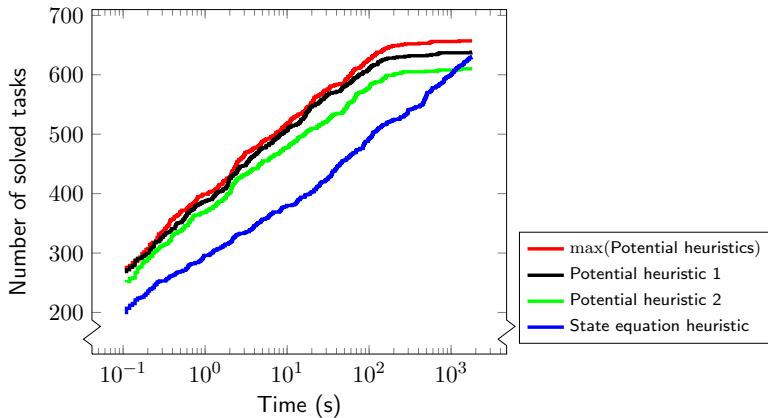
Theorem

Potential heuristic optimized in each state
=
State equation heuristic

Optimizing potentials *less frequently*

- Trade off accuracy for evaluation speed
- Here: optimize *once* for heuristic value of initial state

3) Potential Heuristics: Practice



Take Home Messages

Heuristic combination

$$\begin{array}{c} \text{Operator counting} \\ \approx \\ \text{Optimal general cost partitioning} \end{array}$$

Equivalent heuristics

$$\begin{array}{c} \text{State equation heuristic} \\ = \\ \text{Optimal general cost partitioning of atomic projections} \\ = \\ \text{Potential heuristic (optimized in each state)} \end{array}$$

Interesting new heuristic family: **potential heuristics**

Potential Heuristics (Details)

Potential heuristic

Maximize $f(\text{Potentials})$ subject to

$$\sum_V \text{Potential}_{\text{goal}[V]} \leq 0$$

$$\sum_V (\text{Potential}_{\text{pre}(o)[V]} - \text{Potential}_{\text{eff}(o)[V]}) \leq \text{cost}(o) \quad \text{for each } o \in O$$

Heuristic properties

- **Admissibility:** $h(s) \leq h^*(s)$ for all states s
- **Consistency:** $h(s) \leq h(s') + c(o)$ for all transitions $s \xrightarrow{o} s'$
- **Goal awareness:** $h(s) \leq 0$ for all goal states s
- **Goal awareness + consistency** \Leftrightarrow **admissibility + consistency**